Closing Wed: HW_7A, 7B, 7C (7.8, 8.1)
Midterm 2 is Thursday, May 19
Covers: 6.4, 6.5, 7.1-7.5, 7.7, 7.8, 8.1

7.7 Improper Integrals (continued)

Evaluate

1. $\int_{0}^{\infty} e^{5x}\,dx$

2. $\int_{1}^{\infty} \frac{1}{x}\,dx$

Limits Refresher

1. If stuck, plug in values "near" t.
2. Know your basic values (and basic functions)

$$\lim_{t\to\infty} \frac{1}{t^a} = 0, \qquad \text{if } a > 0.$$

$$\lim_{t\to\infty} \frac{1}{e^{at}} = 0, \qquad \text{if } a > 0.$$

$$\lim_{t\to\infty} t^a = \infty, \qquad \text{if } a > 0.$$

$$\lim_{t\to\infty} \ln(t) = \infty.$$

$$\lim_{t\to 0^+} \ln(t) = -\infty.$$

3. If you have an indeterminant form, use algebra and/or L'Hopital's rule

Example:

$$\lim_{t\to\infty} \frac{\ln(t)}{t} =$$

$$\lim_{t\to\infty} t^2 e^{-3t} =$$

Discontinuities in Integrals

Consider

1. $\int_0^2 \frac{1}{\sqrt{x}}\,dx$

2. $\int_0^1 \frac{x}{x-1}\,dx$

3. $\int_0^4 \frac{1}{x-2}\,dx$

Aside**: A small note on **comparison

Suppose you have two functions f(x) and g(x) such that **0 ≤ g(x) ≤ f(x)** for all values of x.

1. If $\int_a^{\infty} f(x)dx$ converges,

   then $\int_a^{\infty} g(x)dx$ converges.

2. If $\int_a^{\infty} g(x)dx$ diverges,

   then $\int_a^{\infty} f(x)dx$ diverges

Example:

Consider

$$\int_2^{\infty} \frac{1}{\sqrt[3]{x^2-1}}dx$$

We don't have a nice way to integrate, but we can determine if this converges or diverges!

Observe $\sqrt[3]{x^2-1} < x^{\frac{2}{3}}$ (why?)

Thus, $\frac{1}{x^{2/3}} < \frac{1}{\sqrt[3]{x^2-1}}$ (why?)

So

$$\int_2^{\infty} \frac{1}{\sqrt[3]{x^2}}dx < \int_2^{\infty} \frac{1}{\sqrt[3]{x^2-1}}dx$$

Application Quick Review

1. Acceleration, velocity, distance
2. Finding Areas
3. Finding Volumes (washers or shells)
4. Average value = $\frac{1}{b-a}\int_a^b f(x)dx$
5. Work = $\int_a^b (Force)(Dist)$

(a) If f(x) = "force formula at x", then

Force = f(x), *Dist* = Δx: $\text{work} = \int_a^b f(x)dx$

(Spring, leaky bucket, …)

(b) *Chain/Cable*: k = force/length

If you label top: x = 0, then

Force = k Δx, *Dist* = x, $\text{work} = \int_a^b k \text{ x } dx$

(c) *Pumping*: k = force/volume

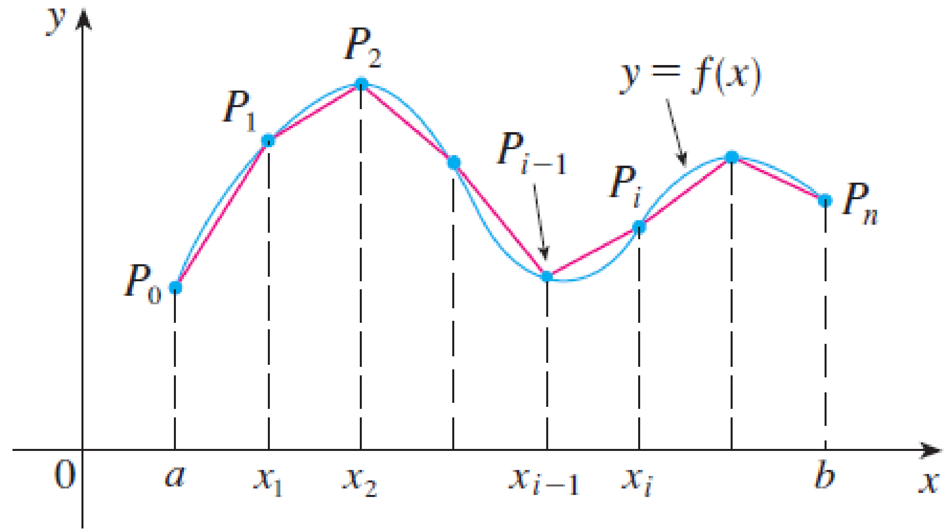
If bottom is y = 0 and top is y = b,

Force = k(Area)Δy, *Dist* = b – y

$$\text{work} = \int_a^b k(\text{Area})(b-y)dy$$

8.1 Arc Length

Goal: Given y = f(x) from x = a to x = b. We want to find the ***length*** along the curve. To approximate we:



1. Break into *n* subdivision:

   $$\Delta x = \frac{b-a}{n}, \qquad x_i = a + i\Delta x$$

2. Compute $y_i = f(x_i)$.
3. Compute the straight line distance from (x_i, y_i) to (x_{i+1}, y_{i+1}).

$$\sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} = \sqrt{(\Delta x)^2 + (\Delta y_i)^2}$$

$$= \sqrt{(\Delta x)^2 \left(1 + \frac{(\Delta y_i)^2}{(\Delta x)^2}\right)}$$

$$= \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x}\right)^2}\ \Delta x$$

4. Add these distances up.

$$\text{Arc Length} = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x}\right)^2}\ \Delta x$$